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General Model of Tool Path Problem for the CNC Sheet Cutting Machines

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Abstract: The formalization of the tool path problem for the CNC sheet metal/material cutting equipment is considered. General model of tool path problem for laser/plasma/gas/water-jet machines is offered. Model uses the term "the basic cutting segment" proposed by author. The existing classification of the tool path problem is expanded. In the paper also the discretization of offered general statement of optimization problem is described. Results of computing experiments for some instances are given.

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Keywords: Tool path problem, CNC sheet cutting machines, general optimization model, basic cutting segment, Segment Continues Cutting Problem (SCCP), discretization of model, GTSP, ICP, Generalized SCCP, "dynamic" constraints

INTRODUCTION

Tool path (Cutting path) problem for CNC machines is one of the most complex optimization problems in Computer Aided Manufacturing. CAM systems are using for generation NC programs for different CNC equipment. Optimization of machining parameters for cutting process provides in particular the minimizing of machining time and of others cost optimization criterions. At present, in various industries for cutting sheet material CNC laser/plasma/gas/water-jet machines are actively used. Despite the difference between these CNC sheet cutting machines, the tool path always contains the following common main components, regardless of the cutting technology, features of equipment and material for cutting (see, in particular, Petunin and Stylios (2016)):

- pierce points (piercings) of sheet material by tool;
- points of switching the tool off;
- tool trajectory from piercing up to point of switching the tool off (the cutting segment);
- Airtime motions/ idling tool path (linear movement from tool switching off point up to the next piercing).

In the cutting segment, there are usually two sub-segments (see Fig 1.):

- lead-in (tool trajectoryy from piercing upto the entry point on the contours, that are the boundaries of parts);
- lead-out (tool trajectoryy from exit point on contour up to points of switching the tool off).

In some cases, the point of switching the tool off may be directly on the contour, i.e. the lead-out length equals zero. The length of led-in is always greater than zero. This is because the diameter of the piercing zone is greater than the width of the cut. There are three main numerical parameters that determine the value of the objective function (optimization criterion) in the tool path optimization:
1. The length of the tool work path;
2. Length of the tool idling path;
3. The number of pierce points.
The remaining parameters, as a rule, are constants and are determined by the specific conditions of the problem.

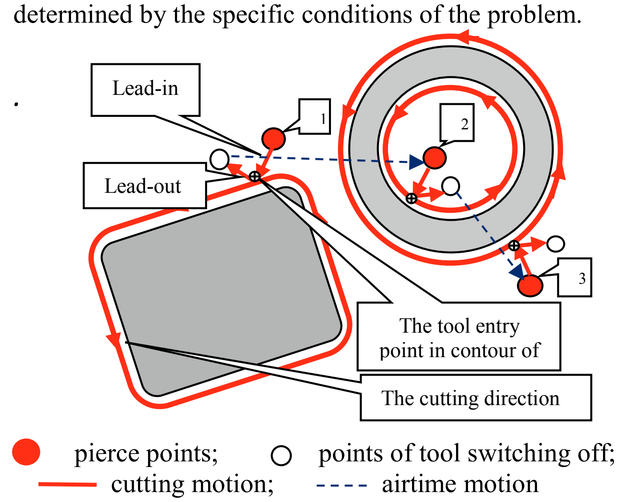

Figure 1. The cutting scheme example for two parts using standard cutting technique

Note that in the Fig. 1 the tool path is visualized not along the boundary contours of the parts, but along their equidistant contours, since the cutting trajectory to maintain the geometrical dimensions of the cut out parts must be half the cut width from the boundary contours.

A feature of the tool path problem (routing problem) for sheet cutting machines is the lack of a general mathematical formalization of the problem. In the scientific literature, when describing algorithms for solving the tool path problem for CNC sheet cutting machines, a narrow class of problems is usually considered. Perhaps the first classification of some types of optimization tasks was given by Hoeft and Palekar (1997). They described, in particular, 3 main classes of tasks: Continuous Cutting Problem (CCP), Endpoint Cutting Problem (ECP) and Intermittent Cutting Problem (ICP). A detailed overview of

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the existing classification of the Cutting path problem is given by Dewil et al. (2015a, 2016).

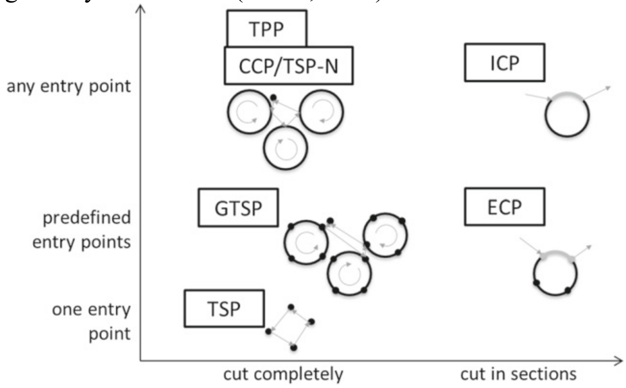

Figure 2. Different versions of Cutting path problem (Fig.6 from paper of Dewil et al. (2016)).

In fact, the difference in classes of problem is determined by the difference in the rules for selecting the possible pierce points or the possible tool entry points in contour of part. The CCP/TSP-N model assumes that the enter points of tool in contour are selected from the continual set of points contained in contour of parts. Some algorithms of solution for CCP are developed by Gentilini et al. (2013), Alatartsev et al. (2013), Alatartsev and Ortmeier (2014) Vicencio et al. (2014).

The other classes restrict the choice of entry points to a finite set. Modern classification includes, in particular, various versions of Traveling Salesman Problem (TSP) and Generalized Traveling Salesman Problem (GTSP). The TSP model is trivial and irrelevant when solving the problem of a tool path generation for CNC sheet cutting machines. The results obtained by Afiffi et al. (2010, 2012) and Erdos et al. (2013) are purely theoretical. Quite a lot of publications describe the use of the GTSP model (see, in particular Yang et al. (2010), Jing and Zhige (2013), Yu and Lu (2014), Chentsov, A. G., and Salii (2015) Chentsov, A.A. and Chentsov, A.G. (2013), Chentsov, P.A. (2014), Helsgaun (2014)). ECP is investigated, in particular, by Kolakowska (2014) and Dewil et al. (2011, 2014, 2015b).

The cutting technique used has a significant effect on the tool path. So, if only so-called the standard cutting technique is used, i.e. each contour is cut out entirely with the help of one pierce point, in this case we get the CCP or its discrete analogue: GTSP. If, however, a multi-segment cutting technique is needed, in which partial cutting of the contour is allowed and a finite set of points for entering the contour is specified, then we have ECP. Petunin (2015) offered one more class of problem: Segment Continuous Cutting Problem (SCCP). This class is the expanding of CCP. It assumes that each segment of a tool path may contain several contours (the multi-contour cutting technique) or, on the contrary, a part of the contour. SCCP is easily reduced to discrete mathematical model GTSP.

It is necessary to separate GTSP (see Fig.2) as a discrete version of the CCP and the mathematical model GTSP, which can be used for solving any cutting path problems: GTSP, CCP, ECP and SCCP. For discrete variant of SCCP Petunin (2015) and Chentsov A.G. et al. (2018) have developed an exact solution algorithm based on the dynamic programming method.

In practice, the user of CAM system has the ability to apply different cutting techniques in interactive mode for any nesting result. However, the existing algorithms for the tool path generation are usually focused on one of above described classes. Below we formulate an approach that allows us to formalize the tool path problem in the most general form (as ICP) and apply various solution algorithms oriented to different classes of problem.

BASIC DEFINITIONS AND CONCEPTS

To formalize the problem of optimizing the movement of the tool at a CNC sheet cutting machine, we introduce the following notation. Let $A_1, A_2, ..., A_n$ to be finite set of two-dimensional geometrical objects. These objects are geometrical models of flat parts. Each object is described by one or several closed curves (boundary contours). Let also N to be number of external and the internal contours $C_1, C_2, ..., C_N$ that describe parts positions (the nesting) on sheet material. $(A_i, C_j \subset \mathbb{R}\times\mathbb{R}; i=\overline{1,n}; j=\overline{1,N}; N \geq n)$

Let also the objects placement area B is defined, which is a geometrical model of sheet material from which details are cut. In general, the placement area may contain several pieces of material (not necessarily rectangular in shape), but to solve the optimization problems of tool routing, it is advisable to consider as a placement area one area bounded by one outer contour. In this case, the presence of holes in the material (internal contours) can be too. We will assume that a certain variant of the placement of objects in the placement area is fixed, while the conditions of mutual non-intersection of objects are fulfilled. We also believe that other additional conditions are met, due to the technological requirements of cutting parts on a specific CNC process equipment. In other words, the fixed positions for placing objects (nesting result) is a valid option for cutting of a given set of parts. Example of nesting is shown in Fig. 3 (n=14, N=21)

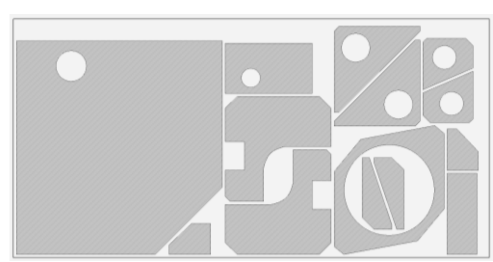

Figure 3. Example of nesting for sheet 2000x1000 mm

Definition 1. Segment of cutting (the cutting segment) $\vec{S} = \overrightarrow{MM^*}$ is a tool trajectory from piercing M up to point of switching the tool off M^*.

$(S \subset \mathbb{R}\times\mathbb{R}; M=(x,y), M^* = (x^*, y^*) \in \mathbb{R}\times\mathbb{R})$

Geometrically, the cutting segment is a curve defined on the Euclidean plane. We will also assume that at each point of the trajectory the direction of movement of the tool is determined. Note that if the cutting segment does not contain closed contours, then the direction of the cutting movement at each point of the trajectory is uniquely determined by the starting point of the segment (the pierce point). Closed contours in the tool path can appear not only as a result of cutting the contours of the parts, but also when programming so-called loops that are used to improve the quality of the cut.

For example, loops in a segment may appear when programming a so-called "sharp angle" or using special cutting techniques.

On Fig. 4 Example of cutting two parts containing three contours (Fig. 1) by using two segments is showed. For cutting the outer contours of parts used so-called "Chain cutting" with one piercing.

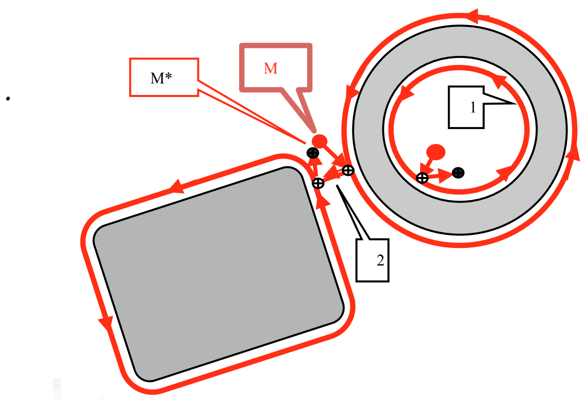

Figure 4. Example of using two segments for cutting of three contours

As we showed above in Fig.1 visualization of the tool path is executed along equidistant contours. At the same time, in most CAM systems, tool movement programming is initially carried out along the boundary contours of parts, and the calculation of the real trajectory is performed either directly by the CNC system itself, or through a special postprocessor program designed to convert information about the tool path from the internal format of the system to the command format specific process equipment with CNC.

In the first case, the allowance for the cut is set by the operator on the machine before running the cutting control program. In the future, without loss of generality, we will assume that the tool path is programming along the boundary contours, and the tool path itself contains all the boundary contours of the parts.

Let K be the number of segments the tool path consists of. $\vec{S_k} = \overrightarrow{M_k M_k^*}; k = \overline{1,K}$. Single segment may contain one contour, a few contours (for the multi-contour cutting), or a part of contour (for multi-segment cutting). Sequence of segments is a permutation $i_1, i_2, \ldots, i_K$, i.e. the ordered set of natural numbers from 1 to K or bijection on a set $\{\overline{1,K}\}$

Thus, the tool path is defined in terms of the cutting segments by a tuple:

$$ROUTE = \langle M_0, M_1, \vec{S_1}, M_1^*, \ldots, M_K, \vec{S_K} M_K^*, i_1, \ldots, i_K \rangle \quad (1)$$

Here M_0 is initial point of tool. We introduce another definition.

Definition 2. Basic segment $B^S, j = \overline{1,N}$ is a part of segment $\vec{S} = \overrightarrow{MM^*}$ without lead-in trajectory and lead-out trajectory.

Let's consider that unlike a cutting segment the corresponding basic segment has no direction of cutting, i.e. it contains only geometry information.

An Example of the segment of cutting including basic segment using complex cutting technique "common cut" is shown in Figure 5.

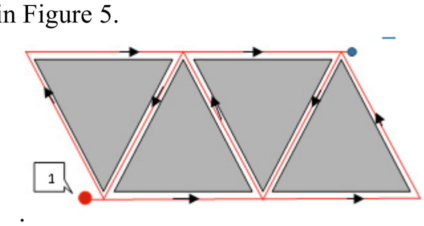

Figure 5. The cutting segment including basic segment with a "common cut"

In Fig. 6 (see also Fig.3.) eight (8) basic segments are defined by outer boundary contours of grey parts. Three (3) additional basic segments are defined by six (6) outer boundary contours of the color parts (one basic segment consists two outer contours plus bridge between them. These contours will be cut by chain cutting pairwise in one cutting segment. At last, seven (7) basic segments are allocated with inner boundary contours of the all parts that have the holes. Total, in this case we have eighteen (18) cutting segments. All basic segments are highlighted in red.

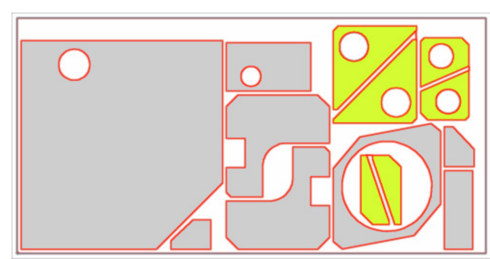

Figure 6. Illustration of term "basic segment" for an Example given in Fig. 3

Naturally, as the set of base segments for this example all boundary contours of parts also can be selected, i.e. $C_j = B^{S_j}, j = \overline{1,N}, K = N = 21$

In this case we will have classical CCP task.

By definition $\vec{S_j} = \overrightarrow{\langle lead_{in}{}^j, B^{S_j}, lead_{out}{}^j \rangle}$ but since the basic segment has no direction, then when basis segment contains one or more closed contours, direction of cutting ("+" for clockwise, "-" for counter clockwise) must be specified for each of them.

Every basic segments contains list $L(B^{S_j})$) of its closed contours (may be empty). Let $|L(B^{S_j})|$ is length of this list. Then

$$\vec{S_j} = \langle lead_{in}{}^j, B^{S_j}, p_j^1, \ldots p_j^r, \ldots p_j^{|L(B^{S_j})|}, lead_{out}{}^j \rangle, \quad (2)$$

where $p_j^r = \pm 1$ - cutting direction, $j = \overline{1,K}$.

Formulas (1) and (2) give the most general mathematical definition of a cutting path (route) for CNC sheet cutting machines.

3. GENERAL STATEMENT OF THE TOOL PATH PROBLEM

Let G – Set of all admissible the cutting paths $ROUTE$ for given placement parts $A_1, A_2, ..., A_n, i = \overline{1,n}$ in area *B*. Let also on this set be defined some objective function *F*. The value of this objective function is most often defined by 3 parameters: length of the work path L_{on}, length of the idling path (airtime move) L_{off}, number of pierce points N_{pt}. All these parameters have been defined by *ROUTE,* i.e. $F(ROUTE) = F(L_{off}, L_{on}, N_{pt})$

Other parameters are constants. In particular, cost of the cutting process for the sheet cutting machines is calculated by

$$F = L_{off} * C_{off} + L_{on} * C_{on} + N_{pt} * C_{pt} \quad (3)$$

Here C_{off} is cost of idling tool path unit; C_{on} is cost of working tool path unit; C_{pt} is cost of one piercing. Thus, the cutting path optimization problem can be compiled on base the term of basic cutting segments as follows:

$$F(ROUTE) = F(L_{off}, L_{on}, N_{pt}) \to \min, ROUTE \in G, \quad (4)$$

where $ROUTE$ is defined by (1) and (2):

$$ROUTE = \langle M_0, M_1, \overrightarrow{S_1}, M_1^*, ..., M_K, \overrightarrow{S_k} M_K^*, i_1, ..., i_K \rangle$$

$$\overrightarrow{S_j} = \langle lead_{in}{}^j, B^{S_j}, p_j^1, ... p_j^r, ... p_j^{|L(B^{S_j})|}, lead_{out}{}^j \rangle$$

There are two types of constraints for admissible tool path: “static” and “dynamic” one.

1. First type of static constraints defines the allowable areas for pierce points $M_j \;\; j = \overline{1,N}$ in the placement area *B*. Other of static constraints is so called precedence constraints. It defines admissible permutations $i_1, i_2, ..., i_K$ in tuple $ROUTE$. Both types of constraints (see, for Examples, Petunin (2015)) are well studied and are not difficulty in the development of cutting path generation algorithms. Moreover, in many works of Prof. Chentsov it was shown that the precedence constraints substantially reduce the time for searching of optimal permutation $i_1, ..., i_K$ for different routing problems.

2. The greatest difficulty is dynamic constraints. This kind of constraints depends on «history» of cutting path construction. Two types of such constraints: the “part hardness rule” and the “sheet hardness rule” are described in Petunin (2009) and Petunin and Stylios (2016). These constraints are caused by the fact that in the process of thermal cutting of metal there can be thermal deformation of the material and distortion of the geometry of parts. Currently, there are only a few publications on this topic of Han et al. (1999), Han and Na (1999), Kim et al. (2004), Dewil (2012), However, these papers do not give the mathematical formalization of constraints generated by thermal effects. At present, only one dynamic constraint defined the choice of pierce points $M_j \;\; j = \overline{1,N}$ (“part hardness rule”) has been formalise by P.A.Chentsov and A.A.Petunin (2016). Note that dynamic constraints for the other routing problem with the costs depend on the set of pending tasks have been formalised by A.A.Chentsov and A.G.Chentsov (2013) and A.G. Chentsov and Ya. V. Salii (2015).

Note also that the cardinality of a set G of admissible of cutting path $ROUTE$ in the general case is a continuum. Below we consider some finite subsets G^{fin} of G. Let a fixed set of the basic segments has been defined for problem (4) with objective function (3).

Definition 3: SCCP (Segment Continuous Cutting Problem) is a problem with fixed number *K* and fixed set of basic segments $B^{S_k}\; k = \overline{1, K}$.

Note: If all contours of parts $C_1, C_2, \; ...C_N; j = \overline{1, N}$ are the basic segments $B^{S_k}, k = \overline{1, K}$ and *N*=*K* then SCCP is equivalent CCP.

Suppose that for the initial problem a finite set of the basic segments sets is fixed (an ensemble of base segments of dimension *T*). This ensemble corresponds to ensemble of $SCCP_i, i = \overline{1,T}$.

Definition 4: GSCCP (Generalized SCCP) is $\{SCCP_i \mid i = \overline{1,T}\}$

Thus, by introducing the class of GSCCP, we have significantly expanded the existing classification of tool path problem for CNC sheet cutting machines. Actually SCCP and GSCCP are subclasses ICP containing all tasks with finite sets of basic cutting segments, i.e. $CCP \subset SCCP \subset GSCCP \subset ICP$. As the author’s best knowledge, no attempt has been made to solve the tool path problem as ICP.

Thus, in ICP class has been selected the big class of cutting path problems for which it is possible to develop the effective optimization algorithms.

4. DISCRETIZATION OF GSCCP. COMPUTING EXPERIMENTS

It is easy to see that each SCCP can be solved using various algorithms, including using models of discrete optimization. In this section, we will give an example of the reduction of GSCCP to the problem of sequential bypassing the megalopolises of Prof. Chentsov (see, for Example, Chentsov et al. (2014)). This problem is reduced to mathematical model GTSP with additional constraints.

In contrast to the classic GTSP this model provides accounting so-called internal work. In addition, model through the using of a special dynamic programming scheme takes into account complex types of objective functions and complex constraints, including dynamic ones. In addition, by taking into account the precedence constraints it is possible to obtain exact solutions for discrete variants of SCCP.

As an example, consider the GSCCP task, which contains two (2) SCCP tasks with the sets of segments shown in Figure 3 and Figure 6 (21 and 18, respectively). As objective function, a popular function is chosen – cutting (machining) time. For this function, the constants in formula (3) will have the following meaning:

$$C_{off} = 1/V_{off}, C_{on} = 1/V_{on} +, C_{pt} = t_{pt}$$

where V_{off} is speed of idling tool path; V_{on} is speed of the working tool path; t_{pt} is time of one piercing.

To reduce SCCP to a discrete model each base segment divided with a certain step by points that will be contenders for the tool entry point into the contour.
Each such point is associated with the pierce point. At same time each possible pierce point has to satisfy technological constraints of cutting process, i.e. many points of basic segments will be deleted. Let me remind you that for each base segment such point can be only one. Fig. 7shows the set of possible pierce points for first task (set of twenty one basic segments).

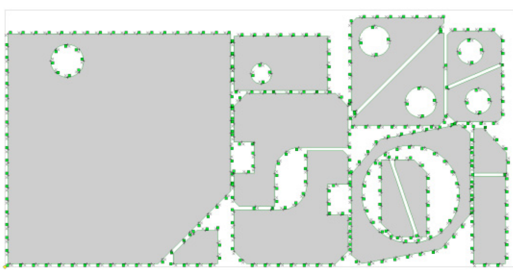

Figure 7. Discretization of CCP task to GTSP for example given in Fig. 3

Actually we reduced CCP task to GTSP model. Note that in this case the number of possible tool paths becomes finite, i.e. $ROUTE \in G^{fin} \subset G$. Figures 8 and 9 are showing the optimal tool paths for tasks with 21 and 18 basic segments.

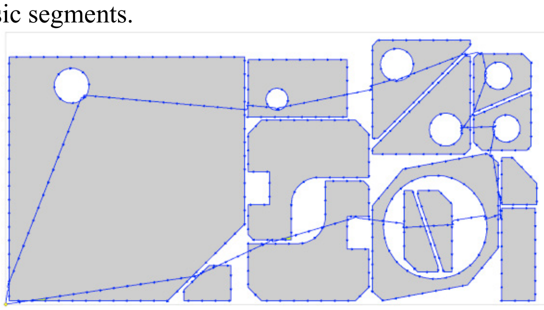

Figure 8. The optimal tool path for 21basic segments

In first case time of cutting process is 2255 sec., in second one is 2244 sec. Note that in the first case, the length L_{on} (20567mm) is less than in the second one (20727mm), but due to the decrease in the number of pierce points for 18 segments, the total cutting time also decreased. Once again, we note that both solutions are optimal for selected sets of base segments. Thus, the optimal value of objective function for selected GSCCP task is 2244 sec.

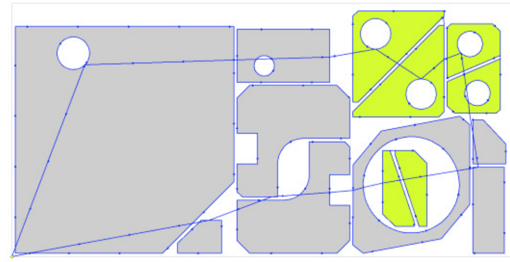

Figure 9. The optimal tool path for 18 basic segments

At solving the tasks, the necessary "static" constraints were taken into account: precedence conditions and constraints for coordinates of piercings. Dynamic constraints in these instances are not considered.

The described approach allow to solve the most difficult class of the tool path routing problems – ICP, that provides no constraints on the choice of the tool entry point in contour of part and using of any cutting technique. The most important feature of the approach is the possibility for one optimization problem (4) to form different sets of base segments and apply different optimization algorithms using both discrete and continuous models.

An obvious development of the proposed approach is the development of methods for generating sets of base segments in the automatic mode of CAM system. Some work in this direction is already underway (see, in particular, Tavaeva et al. (2017), Tavaeva and Kurennov (2015)).

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